Source Coding for Permutation-Invariant Computation

Traditional modes of lossy compression aim to represent a time series or vector with a small number of bits while facilitating reconstruction of an approximate version with low mean-squared error. However, reconstruction of the data itself is not always necessary. Performance might be dramatically improved if the goal is to compute or approximate a function of the data. This is explored in two settings:

1. Encoding of samples without regard to their order is appropriate when the receiver will use the samples in some permutation-invariant computation. In centralized encoding, disregarding order reduces rate requirements for n samples from $O(n)$ to $O(\log n)$. Universal coding results are also obtained.

2. Optimal fixed- and variable-rate distributed quantization of n samples for the computation of a monotonic function of the samples is developed. Several examples with permutation-invariant functions show that using functional form in the quantizer design yields large gains, including distortion reduction exponential in n for the variable-rate case.